IOSR Journal of Engineering (IOSRJEN)
e-ISSN: 2250-3021, **p**-ISSN: 2278-8719
Vol. 3, Issue 1 (Jan. 2013), ||V1|| PP 70-74

Decomposition Of β-Closed Sets In Supra Topological Spaces

S. Dayana Mary[1] and N. Nagaveni[2]
[1,2] *(Department of Mathematics, Coimbatore Institute of Technology, India)*

Abstract: - In this paper, we introduce a new class of sets called supra β- locally closed sets and new class of maps called supra β-locally continuous functions. Furthermore, we obtain some of their properties.

Keywords: - *S-β-LC sets, S-β-LC* sets, S-β-LC** sets, S-β-L-continuous and S-β-L-irresolute.*

I. INTRODUCTION

Njastad [1] defined and studied β-sets in topological spaces. Bourbaki [2] defined a subset of space (X, τ) is called locally closed, if it is the intersection of an open set and a closed set. In topological space, some classes of sets namely generalized locally closed sets were introduced and investigated by Balachandran et al. [3]. The notion of β-locally closed set in topological spaces was introduced by Gnanambal and Balachandran [4]. Mashhour et al. [5] introduced the supra topological spaces and studied S-continuous functions and S*-continuous functions. Ravi et al. [6] introduced and studied a class of sets and maps between topological spaces called supra β-open sets and supra β-continuous maps, respectively. Dayana Mary [7] introduce a new class of sets called supra generalized locally closed sets and new class of maps called supra generalized locally continuous functions. They also introduce a new class of sets called supra regular generalized locally closed sets [8] and S-RGL-continuous functions.

In this paper we introduce the concept of supra β-locally closed sets and study its basic properties. Also we introduce the concepts of supra β-locally continuous maps and investigate several properties for these classes of maps.

II. PRELIMINARIES

Throughout this paper, (X, τ), (Y, σ) and (Z, η) (or simply, X, Y and Z) represent topological space on which no separation axioms are assumed, unless explicitly stated. For a subset A of (X, τ), cl (A) and int (A) represent the closure of A with respect to τ and the interior of A with respect to τ, respectively. Let P(X) be the power set of X. The complement of A is denoted by X-A or A^c.

Now we recall some Definition:s and results which are useful in the sequel.

Definition:: 2.1 [5,9]

Let X be a non-empty set. The subfamily $\mu \subseteq P(X)$ is said to a supra topology on X if $X \in \mu$ and μ is closed under arbitrary unions. The pair (X, μ) is called a supra topological space.

The elements of μ are said to be supra open in (X, μ). Complement of supra open sets are called supra closed sets.

Definition:: 2.2 [9]

Let A be a subset (X, μ). Then

(i) The supra closure of a set A is, denoted by $cl^{\mu}(A)$, defined as $cl^{\mu}(A) = \cap$ {B : B is a supra closed and $A \subseteq B$}.

(ii) The supra interior of a set A is, denoted by $int^{\mu}(A)$, defined as $int^{\mu}(A) = \cup$ {B : B is a supra open and $B \subseteq A$}.

Definition:: 2.3 [5]

A Let (X, τ) be a topological space and μ be a supra topology of X. We call μ is a supra topology associated with τ if $\tau \subseteq \mu$.

Definition:: 2.4 [10]

Let (X, τ) and (Y, σ) be two topological spaces and $\tau \subseteq \mu$. A function f: $(X, \tau) \rightarrow (Y,\sigma)$ is called supra continuous, if the inverse image of each open set of Y is a supra open set in X.

Definition:: 2.5 [11]

Let (X, τ) and (Y, σ) be two topological spaces and μ and λ be supra topologies associated with τ and σ respectively. A function f : $(X, \tau) \rightarrow (Y,\sigma)$ is said to be supra irresolute , if f^{-1} (A) is supra open set of X for every supra open set A in Y.

Definition:: 2.6 [6]

Let (X, μ) be a supra topological space. A subset A of X is called supra β-open if $A \subseteq cl^{\mu}(int^{\mu}(cl^{\mu}(A)))$.

The complement of supra β-open set is called supra β-closed. The class of all supra β-open sets is denoted by S-βO(X)

Definition:: 2.7 [6]

Let A be a subset (X, μ). Then

(i) The supra β-closure of a set A is, denoted by $cl^{\mu}_{\beta}(A)$, defined as $cl^{\mu}_{\beta}(A) = \cap$ {B : B is a supra β-closed and $A \subseteq B$}.

(ii) The supra β-interior of a set A is, denoted by $int^{\mu}_{\beta}(A)$, defined as $int^{\mu}_{\beta}(A) = \cup$ {B : B is a supra β-open and $B \subseteq A$}.

III. SUPRA β-LOCALLY CLOSED SETS

In this section, we introduce the notions of supra β-locally closed sets and discuss some of their properties.

Definition:: 3.1

Let (X, μ) be a supra topological space. A subset A of (X, μ) is called supra β-locally closed set (briefly supra β-LC set), if A=U ∩ V, where U is supra β-open in (X, μ) and V is supra β-closed in (X, μ).

The collection of all supra generalized locally closed sets of X will be denoted by S-β-LC(X).

Remark: 3.2

Every supra β-closed set (resp. supra β-open set) is S-β-LC.

Definition: 3.3

For a subset A of supra topological space (X, μ), A ∈ S-β-LC*(X, μ), if there exist a supra β-open set U and a supra closed set V of (X, μ), respectively such that A=U ∩ V.

Definition: 3.4

For a subset A of (X, μ), A ∈ S-β-LC**(X, μ), if there exist an supra open set U and a supra β-closed set V of (X, μ), respectively such that A=U ∩ V.

Definition: 3.5

Let (X, μ) be a supra topological space. If the space (X, μ) is called a supra B-space, then the collection of all supra β-open subsets of (X, μ) is closed under finite intersection.

Example 3.6

Let X = {a, b, c, d} and μ = {ϕ, X, {a}, {a, b}, {b, c}, {a, b, c}}. Then S-βO(X) = { ϕ, X, {a}, {b}, {a, b}, {a, c},{a, d}, {a, b, c}, {a, b, d},{a, c, d}}. Hence (X, μ) is supra B-space.

Definition: 3.7

Let A, B ⊆ (X, μ). Then A and B are said to be supra β-separated if A ∩ $cl_{\beta}^{\mu}(B)$ = B ∩ $cl_{\beta}^{\mu}(A)$ = ϕ.

Theorem: 3.8

Let A be a subset of (X, μ). If A ∈ S-β-LC*(X, μ) or A ∈ S-β-LC**(X, μ), then A is S-β-LC.

Proof: The proof is obvious by Definition:s and the following example.

Example 3.9

Let X = {a, b, c, d} and μ = {ϕ, X, {a}, {a, b}, {b, c}, {a, b, c}}. Then S-β-LC(X, μ) = S-β-LC*(X, μ) = P(X). S-β-LC**(X, μ) = P(X)-{{a, d}, {a, c, d}}.

Theorem: 3.10

For a subset A of (X, μ), the following are equivalent:

(i) A ∈ S-β-LC*(X, μ).

(ii) A = U ∩ $cl^{\mu}(A)$, for some supra β-open set U.

(iii) $cl^{\mu}(A)$ – A is supra β-closed.

(iv) A∪ [X- $cl^{\mu}(A)$] is supra β-open.

Proof: (i) ⇒ (ii): Given A ∈ S-β-LC*(X, μ)

Then there exist a supra β-open subset U and a supra closed subset V such that A=U ∩V. Since A ⊂ U and A ⊂ $cl^{\mu}(A)$, A ⊂ U ∩ $cl^{\mu}(A)$.

Conversely, $cl^{\mu}(A)$⊂ V and hence A = U ∩ V ⊃ U ∩ ($cl^{\mu}(A)$). Therefore, A = U ∩ $cl^{\mu}(A)$

(ii) ⇒ (i): Let A = U∩ $cl^{\mu}(A)$, for some supra β-open set U. Then, $cl^{\mu}(A)$is supra closed and hence A = U ∩ $cl^{\mu}(A)$ ∈ S-GLC*(X, μ).

(ii) ⇒ (iii): Let A = U ∩ $cl^{\mu}(A)$, for some supra β-open set U. Then A ∈ S-β-LC*(X, μ). This implies U is supra β-open and $cl^{\mu}(A)$ is supra closed. Therefore, $cl^{\mu}(A) -$ A is supra β-closed.

(iii) ⇒ (ii): Let U= X – [$cl^{\mu}(A)$- A]. By (iii), U is supra β-open in X. Then A = U ∩ $cl^{\mu}(A)$ holds.

(iii) ⇒ (iv): Let Q =$cl^{\mu}(A)$–A be supra β-closed. Then X-Q = X - [$cl^{\mu}(A)$– A] =A ∪ [(X- $cl^{\mu}(A)$]. Since X-Q is supra β-open, A ∪ [X- $cl^{\mu}(A)$] is supra β-open.

(iv) ⇒ (iii): Let U = A ∪ [(X- $cl^{\mu}(A)$]. Since X – U is supra β-closed and X - U = $cl^{\mu}(A)$ - A is supra β-closed.

Theorem: 3.11

For a subset A of (X, μ), the following are equivalent:

(i) A ∈ S-β-LC(X, μ).

(ii) A = U ∩ $cl_{\beta}^{\mu}(A)$, for some supra β-open set U.

(iii) $cl_{\beta}^{\mu}(A)$ - A is supra β-closed.

(iv) A ∪ [X - $cl_{\beta}^{\mu}(A)$] is supra β-open.

(v) A ⊆ int_{β}^{μ}(A∪ [X - $cl_{\beta}^{\mu}(A)$].

Proof: (i) ⇒ (ii): Given A ∈ S-β-LC(X, μ)

Then there exist a supra β-open subset U and a supra β-closed subset V such that A=U ∩V. Since A ⊂ U and A ⊂ $cl_{\beta}^{\mu}(A)$, A ⊂ U ∩ $cl_{\beta}^{\mu}(A)$.

Conversely $cl_{\beta}^{\mu}(A)$ ⊂ V and hence A = U ∩ V ⊃ U ∩ $cl_{\beta}^{\mu}(A)$. Therefore A = U ∩ $cl_{\beta}^{\mu}(A)$.

(ii) ⇒ (i): Let A = U∩ $cl_{\beta}^{\mu}(A)$, for some supra β-open set U. Then we have, $cl_{\beta}^{\mu}(A)$ is supra β-closed and hence A = U ∩ $cl_{\beta}^{\mu}(A)$ ∈ S-β-LC*(X, μ).

(ii) ⇒ (iii): Let A = U ∩ $cl_{\beta}^{\mu}(A)$, for some supra β-open set U.

Then A ∈ S-β-LC (X, μ). This implies U is supra β-open and $cl_{\beta}^{\mu}(A)$ is supra β-closed. Therefore, $cl_{\beta}^{\mu}(A) -$ A is supra β-closed.

(iii) ⇒ (ii): Let U= X – [$cl_{\beta}^{\mu}(A)$ - A]. By (iii), U is supra β-open in X. Then A = U ∩ $cl_{\beta}^{\mu}(A)$ holds.

(iii) ⇒ (iv): Let Q = $cl^{\mu}_{\beta}(A)$–A be supra β-closed. Then X-Q = X - [$cl^{\mu}_{g}(A)$ – A] =A ∪ [(X- $cl^{\mu}_{\beta}(A)$]. Since X-Q is supra β-open, A ∪ [X- $cl^{\mu}_{\beta}(A)$] is supra β-open.

(vi) ⇒ (iii): Let U = A ∪ [(X- $cl^{\mu}_{\beta}(A)$]. Since X – U is supra β-closed and X - U = $cl^{\mu}_{\beta}(A)$ - A is supra β-closed.

(vi) ⇒ (v): Since U = A ∪ [(X- $cl^{\mu}_{\beta}(A)$] is supra-β-open, A ⊆ int^{μ}_{β}(A ∪ [(X- $cl^{\mu}_{g}(A)$]).

(v) ⇒ (iv): It is obvious.

Theorem: 3.12

Let (X, μ) be a supra B-space and A ⊂ X be S-β-LC. Then

(i) $int^{\mu}_{\beta}(A)$ ∈ S-β-LC(X, μ).

(ii) $cl^{\mu}_{\beta}(A)$ is contained in a supra β-closed set.

(iii) A is supra β-open if $cl^{\mu}_{\beta}(A)$ is supra β-open.

Proof: (i) Let A = U ∩ $cl^{\mu}_{\beta}(A)$, for some supra β-open set U. Now, $int^{\mu}_{\beta}(A) = int^{\mu}_{\beta}\left(U \cap cl^{\mu}_{\beta}(A)\right) = int^{\mu}_{\beta}(U) \cap int^{\mu}_{\beta}\left(cl^{\mu}_{\beta}(A)\right) = int^{\mu}_{\beta}(U) \cap cl^{\mu}_{\beta}\left(int^{\mu}_{\beta}(A)\right)$. Thus $int^{\mu}_{\beta}(A)$ is S-β-LC.

(ii) $cl^{\mu}_{\beta}(A) = cl^{\mu}_{\beta}\left(U \cap cl^{\mu}_{\beta}(A)\right) \subset cl^{\mu}_{\beta}(U) \cap cl^{\mu}_{\beta}(A)$ which is a supra β-closed set.

(iii) $int^{\mu}_{\beta}(A) = int^{\mu}_{\beta}\left(U \cap cl^{\mu}_{\beta}(A)\right) = int^{\mu}_{\beta}(U) \cap int^{\mu}_{\beta}\left(cl^{\mu}_{\beta}(A)\right)$ = U∩$cl^{\mu}_{\beta}(A)$ = A since $cl^{\mu}_{\beta}(A)$ is supra β-open. Hence A is supra β-open.

Theorem: 3.13

If A ⊂ B ⊂ X and B is S-β-LC, then there exists a S-β-LC set C such that A ⊂ C ⊂ B.

Proof: Immediate.

Theorem: 3.14

For a subset A of (X, μ), if A ∈ S-β-LC**(X, μ), then there exist an supra open set G such that A = G ∩ $cl^{\mu}(A)$.

Proof: Let A ∈ S-β-LC**(X, μ). Then A=G ∩ V, where G is supra open set and V is supra β-closed set. Then A =G ∩ V ⇒ A ⊂ G. Obviously, A ⊂ $cl^{\mu}(A)$. ∴ A ⊂ G ∩ $cl^{\mu}(A)$ ----- (1)

Also we have $cl^{\mu}(A)$ ⊂ V. This implies A = G ∩ V ⊃ G ∩ $cl^{\mu}(A)$ ⇒ A ⊃ G ∩ $cl^{\mu}(A)$ ----- (2)

From (1) and (2), we get A = G ∩ $cl^{\mu}(A)$.

Theorem: 3.15

For a subset A of (X, μ), if A ∈ S-β-LC**(X, μ), then there exist an supra open set G such that A = G ∩ $cl^{\mu}_{\beta}(A)$.

Proof: Let A ∈ S-β-LC**(X, μ).

Then A=G ∩ V, where G is supra open set and V is supra β-closed set.

Then A =G ∩ V ⇒ A ⊂ G. Then A ⊂ $cl^{\mu}_{\beta}(A)$. Therefore, A ⊂ G ∩ $cl^{\mu}_{\beta}(A)$ ----- (1)

Also we have $cl^{\mu}_{\beta}(A)$ ⊂ V. This implies, A = G ∩ V ⊃ G ∩ $cl^{\mu}_{\beta}(A)$ ⇒ A ⊃ G ∩ $cl^{\mu}_{\beta}(A)$ ----- (2)

From (1) and (2), we get A = G ∩ $cl^{\mu}_{\beta}(A)$.

Theorem: 3.16

Let A be a subset of (X, μ). If A ∈ S-β-LC**(X, μ), then $cl^{\mu}_{\beta}(A)$ - A supra β-closed and A ∪ [(X- $cl^{\mu}_{\beta}(A)$] is supra β-open.

Proof: The proof is obvious from the Definition:s and results.

Remark 3.17

The converse of the above Theorem: need not be true as seen the following example.

Example 3.18

Let X = {a, b, c, d} and μ = {ϕ, X, {a}, {a, b}, {b, c}, {a, b, c}}. Then {ϕ, X, {a}, {b}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}} is the set of all supra β-closed sets in X and S-β-LC**(X, μ) = P(X) – {a, d} and {a, c, d}. If A = {a, d}, then $cl^{\mu}_{\beta}(A)$ – A = {c} is supra β-closed and A ∪ [(X- $cl^{\mu}_{\beta}(A)$] = {a, b, d} is supra β-open but A ∉ S-β-LC**(X, μ).

Theorem: 3.19

Suppose (X, μ) is a supra B-space. Let A ∈ S-β-LC(X, μ) and B ∈ S-β-LC(X, μ). If A and B are supra β-separated, then A ∪ B ∈ S-β-LC(X, μ).

Proof: Let A∈ S-β-LC(X, μ) and B ∈ S-β-LC(X, μ). By Theorem: 2, there exist supra β-open sets P and S of (X, μ) such that A = P ∩ $cl^{\mu}(A)$ and B = S∩ $cl^{\mu}(B)$. Put L = P ∩ [X - $cl^{\mu}(B)$)] and M = S ∩ [X - $cl^{\mu}(A)$]. Then L ∩ $cl^{\mu}_{\beta}(A)$ = [P ∩ (X- $cl^{\mu}_{\beta}(B)$)] ∩ $cl^{\mu}_{\beta}(A)$= P ∩ $(cl^{\mu}_{\beta}(B))^{c}$ ∩ $cl^{\mu}_{g}(A)$ = A ∩ $(cl^{\mu}_{g}(B))^{c}$ = A, since A ⊂ $(cl^{\mu}_{\beta}(B))^{c}$. Similarly, M ∩ $cl^{\mu}_{\beta}(B)$ = B. Then L ∩ $cl^{\mu}_{\beta}(B)$ = ϕ and M ∩ $cl^{\mu}_{\beta}(A)$ = ϕ. Since X is a supra B-space, L and M are supra β-open. (L ∪ M)∩ L ∩ $cl^{\mu}_{\beta}(A \cup B)$ = (L ∪ M) ∩ ($cl^{\mu}_{\beta}(A)$ ∪ $cl^{\mu}_{\beta}(B)$) = (L ∩ $cl^{\mu}_{\beta}(A)$) ∪ (L ∩ $cl^{\mu}_{\beta}(B)$) ∪ (M ∩ $cl^{\mu}_{\beta}(A)$) ∪ (M ∩ $cl^{\mu}_{\beta}(B)$) = A∪ B. Therefore A ∪ B ∈ S-β-LC(X, μ).

Remark: 3.20

The following is one of the example of the above Theorem:.

Example: 3.21

Consider the example 3.9. Let A = {a} and B = {b}. Then A and B are supra β-separated, because if A ∩ $cl^{\mu}_{\beta}(B)$ = B ∩ $cl^{\mu}_{\beta}(A)$ = ϕ. Then A ∪ B = {a, b}∈ S-β-LC(X, μ).

Definition: 3.22

Let (X, μ) be a supra topological space. A subset A of (X, μ) is called supra β-dense, if $cl^{\mu}_{\beta}(B)$= X.

Definition: 3.23

A supra topological space (X, μ) is called supra β-submaximal, if every supra β-dense subset is supra β-open in X.

Example 3.24

Consider the example 3.9. Here X, {a, b}, {a, b, c} and {a, b, d} are the supra β-dense sets and also supra β-open sets in X. Therefore X is supra β-submaximal.

Theorem: 3.25

A supra topological space (X, μ) is supra β-submaximal if and only if P(X) = S-β-LC(X) holds.

Proof: Necessity: Let A∈P(X) and G= A ∪[X- $cl_{\beta}^{\mu}(A)$]. Then $cl_{\beta}^{\mu}(G)$= and so G is supra β-dense and hence supra β-open by assumption. By Theorem: 3.11, A∈S-β-LC(X). Hence P(X) = S-β-LC(X).

Sufficiency: Let every subset of X be supra β-locally closed. Let A be supra β-dense in X. Then $cl_{\beta}^{\mu}(A)$= X. Now A = A∪[X- $cl_{\beta}^{\mu}(A)$]. By Theorem: 3.11, A is supra β-open. Hence X is supra β-submaximal.

Theorem: 3.26

Let (X, μ) and (Y, λ) be the supra topological spaces.

(1) If M ∈ S- β-LC(X, μ) and N ∈ S- β-LC(Y, λ), then M × N∈S- β-LC(X × Y, μ × λ).

(2) If M ∈ S- β-LC*(X, μ) and N ∈ S- β-LC*(Y, λ), then M × N∈S- β-LC*(X × Y, μ × λ).

(3) If M ∈ S- β-LC**(X, μ) and N ∈ S- β-LC**(Y, λ), then M × N ∈ S- β-LC**(X ×Y, μ × λ).

Proof: Let M ∈ S-SLC(X, μ) and N ∈ S- β-LC(Y, λ). Then there exist a supra semi-open sets P and P' of (X, μ) and (Y, λ) and supra semi-closed sets Q and Q' of (X, μ) and (Y, λ) respectively such that M = P ∩ Q and N = P' ∩ Q'. Then M × N = (P × P') ∩ (Q ×Q') holds. Hence M × N∈S- β-LC(X × Y, μ × λ).

Similarly, the proofs of (2) and (3) follow from the Definition:s.

IV. SUPRA GENERALIZED LOCALLY CONTINUOUS FUNCTIONS

In this section we define a new type of functions called Supra β-locally continuous functions (S-β-L-continuous functions), supra β-locally irresolute functions and study some of their properties.

Definition: 4.1

Let (X, τ) and (Y, σ) be two topological spaces and τ ⊆ μ. A function f : (X, τ) → (Y,σ) is called S-β-L-continuous (resp., S-β-L* - continuous, resp., S-β-L** - continuous), if f^{-1}(A) ∈ S-β-LC (X,μ), (resp., f^{-1} (A) ∈ S-β-LC* (X,μ), resp., f^{-1} (A)∈ S-β-LC** (X,μ)) for each A ∈ σ.

Definition: 4.2

Let (X, τ) and (Y, σ) be two topological spaces and μ and λ be a supra topologies associated with τ and σ respectively. A function f : (X, τ) → (Y,σ) is said to be S-β-L-irresolute (resp., S-β-L*- irresolute, resp., S-β-L**-irresolute) if f^{-1} (A) ∈ S-β-LC (X,μ), (resp., f^{-1} (A) ∈ S-β-LC* (X,μ), resp., f^{-1} (A) ∈ S-β-LC**(X,μ)) for each A ∈ S-β-LC (Y, λ) (resp., A∈ S-β-LC* (Y, λ), resp., A ∈ S-β-LC* (Y, λ)).

Theorem: 4.3

Let (X, τ) and (Y,σ) be two topological spaces and μ be a supra topology associated with τ. Let f: (X, τ) → (Y,σ) be a function. If f is S-β-L*-continuous or S-β-L**-continuous, then it is S-β-L-continuous.

Proof: The proof is trivial from the Definition:s.

Theorem: 4.4

Let (X, τ) and (Y, σ) be two topological spaces and μ and λ be a supra topologies associated with τ and σ respectively. Let f : (X, μ) → (Y, σ) be a function. If f is S-β-L-irresolute (respectively S-β-L* – irresolute, respectively S-β-L**-irresolute), then it is S-β-L-continuous. (respectively S-β-L*-continuous, respectively S-β-L**-continuous).

Proof: By the Definition:s the proof is immediate.

Remark 4.5

Converse of Theorem: 4.3 need not be true as seen from the following example.

Example 4.6

Let X = Y = {a, b, c, d }with τ = {ϕ, X, {a, b, c}}, σ = {{ϕ, Y, {a, b, d}} and μ = {ϕ, X, {a, c}, {b, c}, {a, b, c}, {a, b, d}, {b, c, d}}. Define f : (X, μ) → (Y, σ) by f(a)=a, f(b)=c, f(c)=d and f(d)=b. Here f is not S-β-L**– continuous, but it is S-β-L- continuous. Also f is not S-β-L**-continuous, but it is and S-β-L* - continuous.

Remark 4.7

The following example provides a function which is S-β-L**- continuous function but not S-β-L**- irresolute function.

Example 4.8

Let X= Y = {a, b, c, d} with τ = {ϕ, X, {b, c}, {a, b, c}}, σ = {{ϕ, Y, {a, b, c}}, μ= {ϕ, X, {a, c}, {b, c}, {a, b, c}, {a, b, d}, {b, c, d}} and λ = {ϕ, Y, {a}, {b, c}, {a, b, c}, {a, b, d}}. Let f : (X,μ) → (Y,σ) be the identity map.Here f is not S-β-L*– irresolute, but it is S-β-L*- continuous.

Theorem: 4.9

Let f: (X, τ) → (Y,σ) be supra β-LC-continuous and A be supra β-closed in X. Then the restriction f | A : A → Y is S-β-L-continuous.

Proof: Let U be supra open in Y. Then $f^{-1}(U)$ in supra β-LC in X. So $f^{-1}(U)$ = G∩F where G is supra β-open and F is supra β-closed in X. Now $(f/A)^{-1}(U)$ = (G∩F)∩A = G∩(F∩A) (resp. (G∩A)∩F) where F∩A is supra β-closed (resp. G∩A is supra β-open) in X. Therefore $(f/A)^{-1}(U)$ is supra β-LC in X. Hence f | A is supra β-L-continuous.

Theorem: 4.10

A space (X, μ) is supra β-submaximal if and only if every function having (X, μ) as domain is supra β-L-continuous.

Proof: Necessity: Let (X, μ) be supra β-submaximal. Then β-LC(X) = P(X) by Theorem: 3.25. Let f**:** (X, μ) → (Y, λ) be a function and A ∈σ. Then $f^{-1}(A)$∈S-β-LC(X) and so f is S-β-L-continuous.

Sufficiency: Let every function having (X, μ) as domain be supra β-L-continuous. Let Y= {0, 1} and σ = {ϕ, Y, {0}}. Let A ⊂(X, μ) and f**:** (X, μ) → (Y, λ) be defined by f(x) = 0 if x ∈A and f(x) = 1 if x ∉A. Since f is supra β-L-continuous, A∈S-β-LC(X, μ). Therefore P(X) = S-β-LC(X) and so X is supra β-submaximal by Theorem: 3.25.

Theorem: 4.11

If g: X → Y is S-β-L-continuous and h: Y → Z is supra continuous, then hog : X → Z is S-β-L-continuous.

Proof: Let g: X → Y is S-GL – continuous and h : Y → Z is supra continuous. By the Definition:s, $g^{-1}(V)$ ∈ S-β-LC (X), V ∈Y and h^{-1} (W) ∈Y , W∈ Z. Let W ∈ Z. Then $(hog)^{-1}$ (W) = (g^{-1} h^{-1}) (W) = g^{-1} (h^{-1}(W)) = g^{-1}(V), for V ∈ Y. From this, $(hog)^{-1}$(W) = g^{-1}(V) ∈ S-GLC (X), W ∈ Z. Therefore, hog is S-β-L- continuous.

Theorem: 4.12

If g : X → Y is S-β-L – irresolute and h : Y → Z is S-β-L-continuous , then h o g : X → Z is S-β-L – continuous.

Proof: Let g : X → Y is S-β-L – irresolute and h : Y → Z is S-β-L-continuous. By the Definition:s, g^{-1}(V) ∈ S-β-LC (X) , for V ∈S-β-LC (Y) and h^{-1}(W) ∈S-β-LC (Y) , for W∈ Z. Let W ∈ Z. Then $(hog)^{-1}$(W) = (g^{-1} h^{-1}) (W) = g^{-1}(h^{-1}(W)) = g^{-1}(V), for V ∈ S-β-LC(Y). This implies, $(hog)^{-1}$(W) = g^{-1}(V) ∈ S-β-LC (X), W ∈ Z. Hence hog is S-β-L- continuous.

Theorem: 4.13

If g : X → Y and h : Y → Z are S-β-L – irresolute , then h o g : X → Z is also S-β-L – irresolute.

Proof: By the hypothesis and the Definition:s, we have g^{-1}(V) ∈ S-β-LC (X), for V ∈S-β-LC (Y) and h^{-1}(W) ∈S-β-LC (Y), for W∈ S-β-LC(Z). Let W ∈S-β-LC(Z). Then $(hog)^{-1}$(W) = (g^{-1} h^{-1}) (W) = g^{-1}(h^{-1}(W)) = g^{-1}(V), for V ∈ S-GLC(Y). Therefore, $(hog)^{-1}$(W) = g^{-1}(V) ∈ S-β-LC (X), W ∈ S-GLC(Z). Thus hog is S-GL – irresolute.

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